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class : 21CS4A

Reg.no : 21CSR009

Course : PROBABILITY & STATISTICS

Code : 21MAT45

ASSIGNMENT-4

1. The three samples given below have been obtained from three normal populations with equal variances. Test the hypothesis that the population means are equal at 5%. LOS.

Sample	1	6	8	5	12	9
	2	5	3	8	7	7
	3	10	7	11	10	12

Sol:

Step-1:

H_0 : There's no significant difference between the samples

H_1 : There's a significant difference between the samples

x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
6	5	10	21	36	25	100
8	3	7	18	64	9	49
5	8	11	24	25	64	121
12	7	10	29	144	49	100
9	7	12	28	81	49	144
Total 40	30	50	120	350	196	514

Step-2:

No. of observations, $N = 15$

Step-3:

$T = 120$

Step-4:

CF, $\frac{T^2}{N} = \frac{(120)^2}{15} = 960$

Step-5:

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - \frac{T^2}{N}$$

$$= 350 + 196 + 514 - 960$$

$$= 100$$

Step-6:

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{1600}{5} + \frac{900}{5} + \frac{2500}{5} - 960$$

$$= 320 + 180 + 500 - 960$$

$$= 40$$

Step-7:

$$SSE = TSS - SSC$$

$$= 100 - 40$$

$$= 60$$

Step-8:

ANOVA TABLE

Source of Variation	Sum of Squares	Degrees of freedom	Mean Squares	Variance ratio	Table Value
Between Columns	SSC = 40	$C-1 = 3-1 = 2$	$MSC = \frac{SSC}{C-1} = \frac{40}{2} = 20$	$F_c = \frac{MSC}{MSE} = \frac{20}{5} = 4$	$F_t(2,12) = 3.89$
Error	SSE = 60	$N-C = 15-3 = 12$	$MSE = \frac{SSE}{N-C} = \frac{60}{12} = 5$		

Step-9:

Conclusion:

$F_{cal} \nless F_{tab}$

Hence we reject H_0.

2. Analyse the following Randomised Block Design (RBD - two ANOVA) and find your conclusion.

		Treatments			
		T_1	T_2	T_3	T_4
Blocks	B_1	12	14	20	22
	B_2	17	27	19	15
	B_3	15	14	17	12
	B_4	18	16	22	12
	B_5	19	15	20	14

Origin = 15

	x_1	x_2	x_3	x_4	Total	x_1^2	x_2^2	x_3^2	x_4^2
Y_1	-3	-1	5	7	8	9	1	25	49
Y_2	2	12	4	0	18	4	144	16	0
Y_3	0	-1	2	-3	-2	0	1	4	9
Y_4	3	1	7	-3	8	9	1	49	9
Y_5	4	0	5	-1	8	16	0	25	1
Total	6	11	23	0	40	38	147	119	68

Step-1:

H_0 : There's no significant difference between row means and column means.

H_1 : There's a significant difference between row means and column means.

step-2:

$N = 20$

step-3:

$T = 40$

step-4:

$$CF, \frac{T^2}{N} = \frac{(40)^2}{20} = \frac{1600}{20} = 80$$

step-5:

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}$$

$$= 38 + 147 + 119 + 68 - 80$$

$$= 292$$

Step-6:

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} + \frac{(\Sigma x_4)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{36}{5} + \frac{121}{5} + \frac{529}{5} + 0 - 80$$

$$= 7.2 + 24.2 + 105.8 - 80$$

$$= 57.2.$$

Step-7:

$$SSR = \frac{(\Sigma y_1)^2}{N_2} + \frac{(\Sigma y_2)^2}{N_2} + \frac{(\Sigma y_3)^2}{N_2} + \frac{(\Sigma y_4)^2}{N_2} + \frac{(\Sigma y_5)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{64}{4} + \frac{324}{4} + \frac{4}{4} + \frac{64}{4} + \frac{64}{4} - 80$$

$$= 16 + 81 + 1 + 16 + 16 - 80$$

$$= 50.$$

Step-8:

$$SSE = TSS - SSC - SSR$$

$$= 292 - 57.2 - 50$$

$$= 184.8$$

Step-9: ANOVA Table.

Source of variation	Sum of Squares	Degrees of freedom	Mean squares	Variance ratio	Table Value
Between Columns	$SSC = 57.2$	$C-1 = 4-1 = 3$	$MSC = \frac{SSC}{C-1} = \frac{57.2}{3} = 19.07$	$F_C = \frac{MSC}{MSE} = \frac{19.07}{15.4} = 1.24$	$F_t(3,12) = 3.49$
Between rows	$SSR = 50$	$r-1 = 5-1 = 4$	$MSR = \frac{SSR}{r-1} = \frac{50}{4} = 12.5$	$F_r = \frac{MSE}{MSR} = \frac{15.4}{12.5} = 1.23$	$F_t(12,4) = 5.91$
Error	$SSE = 184.8$	$(C-1)(r-1) = 3\times4 = 12$	$MSE = \frac{SSE}{12} = \frac{184.8}{12} = 15.4$		

Step-10 :

Conclusion :

Cal F_c < Tab F_c ; we accept H_0

Cal F_r < Tab F_r , we accept H_0

3. In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations

B 25	A 18	E 27	D 30	C 27
A 19	D 31	C 29	E 26	B 23
C 28	B 22	D 33	A 18	E 27
E 28	C 26	A 20	B 25	D 33
D 32	E 25	B 23	C 28	A 20

Sol:

Origin = 25

	X_1	X_2	X_3	X_4	X_5	Total	X_1^2	X_2^2	X_3^2	X_4^2	X_5^2
Y_1	0	-7	2	5	2	2	0	49	4	25	4
Y_2	-6	6	4	1	-2	3	36	36	16	1	4
Y_3	3	-3	8	-7	2	3	9	9	64	49	4
Y_4	3	1	-5	0	8	7	9	1	25	0	64
Y_5	7	0	-2	3	-5	3	49	0	4	9	25
Total	7	-3	7	2	5	18	103	95	113	84	101

Step-1:

H_0 : There's no significant difference between row means, column means and treatments.

H_1 : There's a significant difference between row means, column means and treatments.

Step-2:

$N = 25$.

Step-3:

$T = 18$

Step-4:

$$\frac{T^2}{N} = \frac{(18)^2}{25} = \frac{324}{25} = 12.96$$

Step-5:

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 + \Sigma x_5^2 - \frac{T^2}{N}$$

$$= 103 + 95 + 113 + 84 + 101 - 12.96$$

$$= 483.04$$

Step-6:

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} + \frac{(\Sigma x_4)^2}{N_1} + \frac{(\Sigma x_5)^2}{N_1} - \frac{T^2}{N}.$$

$$= \frac{49}{5} + \frac{9}{5} + \frac{49}{5} + \frac{4}{5} + \frac{25}{5} - 12.96$$

$$= 9.8 + 1.8 + 9.8 + 0.8 + 5 - 12.96.$$

$$= \cancel{2.099}\,14.24$$

Step-7:

$$SSR = \frac{(\Sigma y_1)^2}{N_2} + \frac{(\Sigma y_2)^2}{N_2} + \frac{(\Sigma y_3)^2}{N_2} + \frac{(\Sigma y_4)^2}{N_2} + \frac{(\Sigma y_5)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{4}{5} + \frac{9}{5} + \frac{9}{5} + \frac{49}{5} + \frac{9}{5} - 12.96$$

$$= 0.8 + 1.8 + 1.8 + 9.8 + 1.8 - 12.96$$

$$= 3.04.$$

step-8:

SSK:

Arrange the treatments in order.

						Total
A	-7	-6	-7	-5	-5	-30
B	0	-2	-3	0	-2	-7
C	2	4	3	1	3	13
D	5	6	8	8	7	34
E	2	1	2	3	0	8

$$SSK = \frac{(-30)^2}{5} + \frac{(-7)^2}{5} + \frac{(13)^2}{5} + \frac{(34)^2}{5} + \frac{(8)^2}{5} - 12.96$$

$$= 180 + 9.8 + 33.8 + 231.2 + 12.8 - 12.96$$

$$= 454.64$$

step-9:

$$SSE = TSS - SSC - SSR - SSK$$

$$= 483.04 - 14.24 - 3.04 - 454.64$$

$$= 11.12$$

step-10: ANOVA Table.

Source of variation	Sum of Squares	Degrees of freedom	Mean Squares	Variance ratio	Table Value
Between columns	SSC = 14.24	$(k-1) = 5-1=4$	$MSC = \frac{SSC}{k-1} = \frac{14.24}{4} = 3.56$	$F_c = \frac{MSC}{MSE} = \frac{3.56}{0.93} = 3.83$	$F_t(4,12) = 3.26$
Between rows	SSR = 3.04	$(k-1) = 5-1=4$	$MSR = \frac{SSR}{k-1} = \frac{3.04}{4} = 0.76$	$F_r = \frac{MSE}{MSR} = \frac{0.93}{0.76} = 1.22$	$F_t(12,4) = 5.91$

Between treatments	SSK = 454.64	$(K-1)(\cancel{K-2})$ = 4 = 4	MST = $\frac{SSK}{12}$ = $\frac{454.64}{4}$ = ~~113.66~~ = 113.66	$F_t = \frac{MST}{MSE}$ = $\frac{113.66}{0.93}$ = 122.22	$F_t(4,12)$ = 3.26
Error	SSE = 11.12	$(K-1)(K-2)$ = 4×3 = 12	MSE = $\frac{SSE}{12}$ = $\frac{11.12}{12}$ = 0.93		

Step-11:

Conclusion:

cal $F_c \nless$ Tab F_c , we reject H_0

cal $F_r <$ Tab F_r , we accept H_0

cal $F_t \nless$ Tab F_t , we reject H_0.